Dynamic Modeling of a One-wheel Robot by Using Kane's method

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Abstract

In this paper, dynamic modeling of a one-wheel robot, which is subjected to nonholonomic constraints, was derived by using Kane's method. By imposing the constraints in the dynamic equations leads to system order reduction. This method can be applied without using Lagrange multipliers. So the computation complexity is reduced. The motion of the one-wheel robot integrated with a gyroscope for stabilization and steering was described. Numerical simulations are presented to verify validity of the model in agreement with the Lagrange's formulation found in early works by other researchers.

Keywords: One-Wheel Robot, Kane's Method, Gyroscopic Motion, Nonholonomic Constraints

Nomenclatures:

R_w	Radius of disk	q_7	Tilt angle of gyroscope
R_g	Radius of gyroscope	q_8	Spin angle of gyroscope
M	Total mass of robot	u_i	Generalized speeds
m_w	Mass of disk	${}^{A}\boldsymbol{\omega}^{C}$	Angular velocity of frame C wrt. frame A
m_g	Mass of gyroscope	${}^{A}\boldsymbol{\alpha}^{C}$	Angular acceleration of frame C wrt. frame A
m_p	Mass of driving unit exclude gyroscope	${}^{A}\mathbf{P}^{B}$	Position vector of point B wrt. frame A
q_1	Heading angle	${}^{A}\mathbf{v}^{B}$	Linear velocity of point B wrt. frame A
q_2	Leaning angle	${}^{A}\mathbf{a}^{B}$	Linear acceleration of point B wrt. frame A
q_3	Wheel angle	$\mathbf{C}^*$	Mass center of body **C**
q_4	Position of point contact P along $\mathbf{a_x}$	T_1	Leaning torque
q_5	Position of point contact P along $\mathbf{a_y}$	T_2	Rolling torque
q_6	Pendulum angle of m_p wrt. contact point	$T_{1,eq}$	Tilting rate

1. Introduction

Recently, new types of mobile robots which have internal driving mechanisms contained in closed surfaces have been developed. Bicchi [1] and Halme et al. [2] developed spherical profile robots with internal wheels roll on the surface inside the balls. Similar work was done by Bhattacharya and Agrawal [3] but their robot is driven by two perpendicular rotors attached inside the sphere. Brown and Xu [4] proposed a unique wheel-like robot called Gyrover which was actuated by an internal gyroscope mechanism.

This class of mobile robots has many advantages over the conventional mobile robots. They have shells that protect themselves from external environment and are driven by internal mechanisms. Because of their smooth external profiles, they would not be stuck by any obstacles. They also have self-recovery ability after falling when they bump onto some objects. Furthermore they can be used in outdoor environment under various climates. This type of robots will be the alternative solution in exploration robotics. Wheel-like robots have narrow shape than spherical robots. So these robots are more suitable in exploring tasks especially the narrow passages. However, these robots are subject to the nonholonomic constraints while rolling without slipping on horizontal plane.

The one-wheel robot is dynamically stable when speed of its internal gyroscope reaches the conditioning value. The gyroscopic effect is not only stabilizing and balancing the robot, but also steering the robot to track the desired trajectory. Stabilization is maintained in roll and vertical axis against disturbance from irregularity of terrain. Such robotic geometry simplifies motion planning and enhances reliability while moving in faster motion.

Some researchers developed a rolling disk model which can be realized as a simplify version of a one-wheel robot. Rui and McClamroch [5] developed dynamic model of a rolling disk where they assumed that motions in roll, pitch and yaw axes were generated from three decouple torques. Yavin and Frangos [6] showed that only rolling and leaning torques are adequate for controlling a disk moving along the desired path. Xu et al. [7], [8], [9] developed the model of a one-wheel robot as a rolling disk, which had driving mechanism extended from Rui and McClamroch's work.

Most derivations above are based on Lagrange's equation. For nonholonomic constrained systems, Lagrange multipliers will be solved complicatedly. Bloch et al. [10] proposed the elimination of Lagrange multipliers by using matrix partitioning. Alternatively, Kane's equation [11] is used to formulate the dynamic equations concerning nonholonomic constraints. The main advantage of this method is to achieve the equations of motion in term of independent variables without using Lagrange multipliers. Computational complexity can be largely reduced. The equations of motion consist of kinematic equations, dynamic equations, and nonholonomic constraint equations. A set of differential-algebraic equations will be formed and will describe constrained dynamic systems behaviors.

In this paper, we will analyze the motion of a one-wheel robot by using Kane's equation. First, Kane's method and the advantages will be presented. Next, dynamic modeling of the one-wheel robot will be derived. Finally, Numerical simulation and discussion will be provided.

2. Equations of Nonholonomic Constrained Systems

Kane proposed an effective formulation for dynamic modeling of multi-body systems. The generalized coordinates indicate the minimal set of coordinates described the system configurations. For nonholonomic constrained systems, the use of generalized speeds represents the smallest number of velocities variables to describe the possible motions of system. This number is called the degree of freedom of system. Kane's method can impose nonholonomic constraints in the systems which can largely reduce complexity of computations.

2.1 Kane's Method

For a system with *n*-generalized coordinates, $\mathbf{q} \in \mathbb{R}^n$ subjected to m nonholonomic constraints, the generalized speeds, $\mathbf{u} \in \mathbb{R}^n$ can be defined as

$$u_r = \sum_{i=1}^{n} Y_{ri}\dot{q}_i + Z_r \quad (r = 1,\ldots,n) \qquad (1)$$

where $\dot{q}$ is time derivative of q; Y_{ri} and Z_r are functions of q and time t.

The *n*-generalized speeds are composed of a p x 1 unconstrained velocity vector, $\mathbf{u}_s = \begin{bmatrix} u_1 & u_2 & \ldots & u_p \end{bmatrix}^T$ and an m x 1 constrained velocity vector, $\mathbf{u}_c = \begin{bmatrix} u_{p+1} & u_{p+2} & \ldots & u_n \end{bmatrix}^T$ as

$$\mathbf{u} = \begin{bmatrix} \mathbf{u_s} & \vdots & \mathbf{u_c} \end{bmatrix}^T \qquad (2)$$

where $p = n - m$ indicates not only the number of degrees of freedom of the nonholonomic system but also the smallest number of independent generalized speeds which describes the motions of system.

The nonholonomic constraints can be represented as

$$u_c = \sum_{s=1}^{p} A_{cs}u_s + B_c \quad (c = p+1,\ldots,n) \qquad (3)$$

where A_{cs} and B_c are functions of q and time t.

Eq.(3) can be written in the Pfaffian form as

$$\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = 0 \qquad (4)$$

where $\mathbf{A}(\mathbf{q})$ is an m x n full rank constraint matrix.

Assume that $\mathbf{S}(\mathbf{q})$ is an n x p full rank matrix whose subset is a set of smooth and linearly independent vectors in the Null space of $\mathbf{A}(\mathbf{q})$, such we obtain

$$\mathbf{A}(\mathbf{q})\mathbf{S}(\mathbf{q}) = 0 \qquad (5)$$

Imposing nonholonomic constraints in Eq.(3) to the system, Eq.(1) can be solved for each $\dot{q}$,

$$\dot{q}_i = \sum_{s=1}^{p} W_{is}u_s + X_i \quad (i = 1,\ldots,n) \qquad (6)$$

where W_{is} and X_i are functions of q and time t. Eq.(6) are kinematic equations and can be presented in form as

$$\dot{\mathbf{q}} = \mathbf{S}(\mathbf{q})\mathbf{u} \qquad (7)$$

where $\mathbf{S}(q)$ is an n x p full rank, Jacobian matrix that transforms independent velocities $u_1,\ldots,u_p$ to the velocities $\dot{q}$.

Kane's dynamic equations can be represented as:

$$F_r + F_r^* = 0 \qquad (r = 1,\ldots,p) \qquad (8)$$

and

$$F_r = \sum_{i=1}^{N} \mathbf{v}_r^{P_i} \cdot \mathbf{R}_i \qquad (9)$$

$$F_r^* = \sum_{i=1}^{N} \mathbf{v}_r^{P_i} \cdot \mathbf{R}_i^* \qquad (10)$$

where F_r is generalized active/external force and F_r^* is generalized inertia force; $\mathbf{R}_i$ is active force; $\mathbf{R}_i^*$ is inertia force; P_i is the i^{th} particle of system; $\mathbf{v}_r^{P_i}$ is partial velocity of particle P_i with respect to inertial frame; N is the number of particles.

According to Eq.(8), it is equivalent to Newton's second law of motion

$$\mathbf{R}_i - m_i\mathbf{a}_i = 0 \qquad (i = 1,\ldots,N) \qquad (11)$$

where $\mathbf{a}_i$ is acceleration of the i^{th} particle.

The active forces $\mathbf{R}_i$ can be classified as applied forces $\mathbf{R}_i^a$ and constraint forces $\mathbf{R}_i^c$ which can be presented as:

$$\mathbf{R}_i = \mathbf{R}_i^a + \mathbf{R}_i^c \qquad (12)$$

The constraint forces do no work which mean no any movement along the constraint force directions. Such it can be described as:

$$\sum_{i=1}^{N} \mathbf{R}_i^c \cdot \mathbf{v}_r^{P_i} = 0 \qquad (13)$$

As seen in Eq.(13), the constraint forces are eliminated from the dynamic equations. Such the dynamic equations in Eqs.(8), (9) and (10) can be written in a form as

$$\sum_{i=1}^{N} \left(\mathbf{R}_i^a - \mathbf{R}_i^*\right) \cdot \mathbf{v}_r^{P_i} = 0 \qquad (14)$$

Eq.(14) shows that both applied forces and inertia forces are projected onto the motion which is allowed to move [12], [13].

In Eq.(1), one can choose any form of generalized speeds, but the choices of generalized speeds may affect the load of computation especially in large multi-body systems. Mitiguy and Kane [14] proposed the determination of generalized speeds and consequently Jacobian $\mathbf{S}(\mathbf{q})$, that lead to efficient computation of equations of motion.

2.2 Normal Form of Nonholonomic Systems

Dynamic model of constrained systems can be derived by several methods such as Newton-Euler's equation, Lagrange's equation or Kane's method. By using Lagrange's method, the dynamic model will be presented as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = \mathbf{E}(\mathbf{q})\boldsymbol{\tau} + \mathbf{A}^T(\mathbf{q})\boldsymbol{\lambda} \qquad (15)$$

where $\mathbf{M}$ is an n x n mass matrix; $\mathbf{N}$ is an n x 1 a vector of coriolis and centrifugal forces; $\mathbf{G}$ is a n x 1 gravitational force vector, $\mathbf{E}$ is an n x k input coefficient matrix; $\boldsymbol{\tau}$ is a k x 1 external force vector; $\mathbf{A}(\mathbf{q})$ is a constraint matrix which can be expressed in Eq.(4); and $\boldsymbol{\lambda}$ is Lagrange's multipliers vector. The constraint force can be represented as $\mathbf{A}^T(\mathbf{q})\boldsymbol{\lambda}$.

According to Eqs.(5) and (7), we obtain

$$\ddot{\mathbf{q}} = \mathbf{S}\dot{\mathbf{u}} + \dot{\mathbf{S}}\mathbf{u} \qquad (16)$$

$$\mathbf{S}^{\mathrm{T}}\mathbf{A}^T\boldsymbol{\lambda} = 0 \qquad (17)$$

From Eqs.(15), (16) and (17), Lagrange multipliers can be eliminated as

$$\mathbf{S}^{\mathrm{T}}\left(\mathbf{M}\mathbf{S}\dot{\mathbf{u}} + \mathbf{M}\dot{\mathbf{S}}\mathbf{u} + \mathbf{N} + \mathbf{G}\right) = \mathbf{S}^{\mathrm{T}}\mathbf{E}\boldsymbol{\tau} \qquad (18)$$

The system order will be reduced into n-m which is equal to the degrees of freedom of the system.

Kane's method is also used to derive the dynamic equations of m-nonholonomic constraint system. The dynamic equations can be expressed as:

$$\tilde{\mathbf{M}}(\mathbf{q})\dot{\mathbf{u}} + \tilde{\mathbf{N}}(\mathbf{q},\mathbf{u}) + \tilde{\mathbf{G}}(\mathbf{q}) = \tilde{\mathbf{E}}(\mathbf{q})\boldsymbol{\tau} \qquad (19)$$

where $\tilde{\mathbf{M}}$ is an (n-m) x (n-m) mass matrix; $\tilde{\mathbf{N}}$ is an (n-m) x 1 vector of coriolis and centripetal forces; $\tilde{\mathbf{G}}$ is an (n-m) x 1 gravitational force vector; $\tilde{\mathbf{E}}$ is an (n-m) x k input coefficient matrix and τ is a k x 1 external force vector.

As seen in Eq.(19), Lagrange multipliers do not concerned. The nonholonomic constraints are integrated into the dynamic model. This leads to system order and computational complexity reductions.

Eqs.(18) and (19) can be expressed equivalently as:

$$\begin{aligned} \tilde{\mathbf{M}} &= \mathbf{S}^{\mathrm{T}}\mathbf{M}\mathbf{S} \\ \tilde{\mathbf{N}} + \tilde{\mathbf{G}} &= \mathbf{S}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{S}}\mathbf{u} + \mathbf{S}^{\mathrm{T}}[\mathbf{N} + \mathbf{G}] \\ \tilde{\mathbf{E}} &= \mathbf{S}^{\mathrm{T}}\mathbf{E} \end{aligned} \qquad (20)$$

Elimination of the Lagrange multipliers makes the equations of motion concisely. System order is reduced to 2n-m equations. Both Eqs.(18) and (19) are *Normal Form of equations of motion for nonholonomic systems*. Eq.(15) derived by Lagrange's method, n dynamic equations must be solved beforehand. Then the Lagrange multipliers will be eliminated further. Using Kane formulation, Eq.(19), n-m dynamic equations will be solved without the Lagrange multipliers. Obviously, Kane's method is more suitable for dealing with nonholonomic systems.

2.3 State Space Representation

To represent state space form of a nonholonomic systems, kinematic equations, Eq.(7), and dynamic equations, Eq. (19) must be realized. Using Kane's equation, the minimal set of state variables composed of the n-generalized coordinates and n-m generalized speeds can be obtained as shown

$$\begin{aligned} \mathbf{x} &= [\mathbf{q}^T \quad \mathbf{u}^T]^T \\ &= [q_1 \quad q_2 \quad \cdots \quad q_n \quad u_1 \quad u_2 \quad \cdots \quad u_{n-m}]^T \end{aligned} \qquad (21)$$

and state equations can be represented as

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \end{bmatrix}\boldsymbol{\tau} \qquad (22)$$

where $\mathbf{f}_1 = \mathbf{S}\mathbf{u}$; $\mathbf{f}_2 = -\tilde{\mathbf{M}}^{-1}(\tilde{\mathbf{N}} + \tilde{\mathbf{G}})$; $\mathbf{g}_1 = 0$; $\mathbf{g}_2 = \tilde{\mathbf{M}}^{-1}\tilde{\mathbf{E}}$

3. Modeling of a One-Wheel Robot

Considering a rigid disk **C**, with a radius R_W, is rolling on a horizontal plane **H** as shown in Fig. 1. The plane is fixed in inertial reference frame **A**, which formed by a set of unit vectors $\mathbf{a_x}$, $\mathbf{a_y}$ and $\mathbf{a_z}$. The moving reference frame **B**, composed of a set of unit vectors $\mathbf{b_1}$, $\mathbf{b_2}$ and $\mathbf{b_3}$, located at the center of the disk. The vector $\mathbf{b_1}$ is always parallel to the horizontal plane and collinear with heading of the disk. The vector $\mathbf{b_3}$ is a rolling axis of the disk and perpendicular to the disk plane. The disk always contacts with the plane **H** while rolling without slipping.

Driving mechanism is composed of a two links and a gyroscope **G** attached at the end of the second link. It provides driving and leaning torques to control motion of the disk. The first link which has length L_1 is connected to the disk at point $\mathbf{C}^*$, which is the center of the disk. It rotates about the $\mathbf{b_3}$ axis. The second link with length L_2 rotates about the $\mathbf{e_1}$ axis. It is connected between the first link at point W and the gyroscope at point $\mathbf{G}^*$. The gyroscope spins at constant speed about the $\mathbf{f_3}$ axis. Both links are assumed as mass-less. The mass of the driving mechanism is considered as a lump mass m_p at point W.

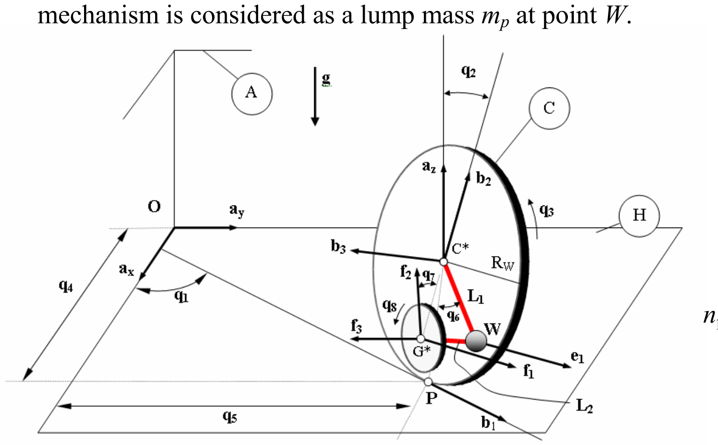

Fig.1 Coordinate Assignment for a One-Wheel Robot Rolling on Horizontal Plane

In order to formulate the equations of motion of a one-wheel robot, we use assumptions appeared in [9] as follows:

1) $L_2 = 0$, so the mass center of the gyroscope is coincident with the mass of the driving mechanism m_p.

2) $q_6 = 0$, Xu et al. [9] observed that the motion of the mass m_p is too small and can be neglected at the steady state.

3) $L_1 = 0$, such the mass center of the gyroscope is coincident with the mass center of the disk

4) Refer to assumption 3, driving torque from the mass m_p will be absent. Such we assume that there is a fictitious driving torque, T_2, applied to the disk

5) The spinning rate of gyroscope $\dot{q}_8$ is set to be constant. Because of the momentum of the gyroscope is too large, it is difficult to change or control its speed.

6) $\dot{q}_7$ is control directly by tilt motor and can be treat as new input, $T_{1,eq} = \dot{q}_7$

Refer to the kinematic equations Eq.(7), and the dynamic equations Eq.(19). The equations of motion of the one-wheel robot are analyzed, as follows:

$$\dot{\mathbf{q}} = \mathbf{S}(\mathbf{q})\mathbf{u}$$

$$\tilde{\mathbf{M}}(\mathbf{q})\dot{\mathbf{u}} + \tilde{\mathbf{N}}(\mathbf{q},\mathbf{u}) + \tilde{\mathbf{G}}(\mathbf{q}) = \tilde{\mathbf{E}}(\mathbf{q})\boldsymbol{\tau}$$

where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 \end{bmatrix}^T$;

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$$

$$\mathbf{S}(\mathbf{q}) = \begin{bmatrix} 0 & \dfrac{1}{\cos q_2} & 0 \\ -1 & 0 & 0 \\ 0 & -\tan q_2 & 1 \\ 0 & R_W \cos q_1 \tan q_2 & -R_W \cos q_1 \\ 0 & R_W \sin q_1 \tan q_2 & -R_W \sin q_1 \end{bmatrix} \tag{23}$$

$$\tilde{\mathbf{M}}(\mathbf{q}) = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \tag{24}$$

$$m_{11} = I_C + I_G + R_W^2 M$$

$$m_{22} = I_C + I_G \left(\frac{1 + \sin^2(q_7 + q_2)}{\cos^2 q_2} \right)$$

$$m_{33} = 2I_C + R_W^2 M$$

$$M = M_P + M_W + M_G$$

$$\tilde{\mathbf{N}}(\mathbf{q},\mathbf{u}) = \begin{bmatrix} n_1 & n_2 & n_3 \end{bmatrix}^T \tag{25}$$

$$n_1 = (2I_C + R_W^2 M)u_2 u_3 + 2I_G \dot{q}_8 \cos(q_7 + q_2)\frac{u_2}{\cos q_2}$$
$$+ I_G \sin(q_7 + q_2)\cos(q_7 + q_2)\frac{u_2^2}{\cos^2 q_2} - I_G \ddot{q}_7 - I_C \tan q_2 u_2^2$$

$$\hat{n}_2 = \mu_s u_2 + I_C \tan q_2 u_1 u_2 - 2I_C u_1 u_3$$
$$- \frac{I_G}{\cos^3 q_2} \left(\begin{array}{l} \sin q_2 u_1 u_2 + \sin q_2 \sin^2(q_7 + q_2) u_1 u_2 \\ +2\dot{q}_8 \cos(q_7 + q_2)\cos^2 q_2 u_1 \\ +2\cos q_2 \sin(q_7 + q_2)\cos(q_7 + q_2) u_1 u_2 \end{array} \right)$$

$$n_3 = u_2(\mu_s \tan q_2 - R_W^2 M u_1)$$

$$\tilde{\mathbf{G}}(\mathbf{q}) = \begin{bmatrix} gMR_W \sin q_2 \\ 0 \\ 0 \end{bmatrix} \tag{26}$$

$$\tilde{\mathbf{E}}(\mathbf{q}) = \begin{bmatrix} 0 & 0 \\ e_{21} & e_{22} \\ 0 & 1 \end{bmatrix} \tag{27}$$

$$e_{21} = \frac{-I_G}{\cos^3 q_2}\left[2\cos q_2 \cos(q_7 + q_2)\left(\dot{q}_8 \cos q_2 + \sin(q_7 + q_2) u_2 \right) \right]$$

$$e_{22} = -\tan q_2$$

$$\boldsymbol{\tau} = \begin{bmatrix} T_{1,eq} \\ T_2 \end{bmatrix} \tag{28}$$

Nonholonomic constraints can be presented as

$$u_4 = R_W \cos q_1 (u_2 \tan q_2 - u_3)$$
$$u_5 = R_W \sin q_1 (u_2 \tan q_2 - u_3) \quad (29)$$

4. Numerical Simulations

A one-wheel robot motion with a leaning angle, q_2=20 degree and a rolling generalized speed, u_3=15 rad/s is simulated.

Parameters: G=9.81 m/s^2, M_W=1.25 kg, M_P=4.4 kg, M_G=2.4 kg, R_W=0.17 m, R_G=0.05 m, μ_S=0.1 Nm.

Initial conditions: q_1=0, q_2=20 deg., q_3=0, q_4=0, q_5=0, q_6=0, q_7=0, u_1=0, u_2=0, u_3 =15 rad/s, $\dot{q}_8$ =1500 rad/s.

The robot is move in a circular path while its lean angle, q_2 is gradually increased. The robot is falling according to friction affect as see in Fig.2, 3, and 4. Fig.2 shows the change of heading angle q_1, leaning angle q_2, and robot position q_4 and q_5. Fig.3 shows time derivative of generalized coordinates, $\dot{q}_i$ and the generalized speeds, u_i. Fig.4 shows position of a contact point (q_4 , q_5) of the robot on X-Y Plane.

We also simulate a rolling disk motion with the same parameters and same initial conditions to compare the results with a one-wheel robot. Because the rolling disk model is well known and its motion very closed to a one-wheel robot motion.

Initial conditions: q_1=0, q_2=20 deg., q_3=0, q_4=0, q_5=0, u_1=0, u_2=0, u_3 =15 rad/s.

In Fig.5, 6, and 7, The disk rolling at the same speed as a one-wheel robot. But its leaning angle, q_2 growth up rapidly and the disk fall down finally.

In leaning motion, the gyroscope within a one-wheel robot produce momentum large enough to stabilize the robot. The gravity torque is also cancel and the robot is in dynamic equilibrium. Both the rolling disk and the one-wheel robot roll faster as their leaning angles increase. This behavior has been analyzed in [15].

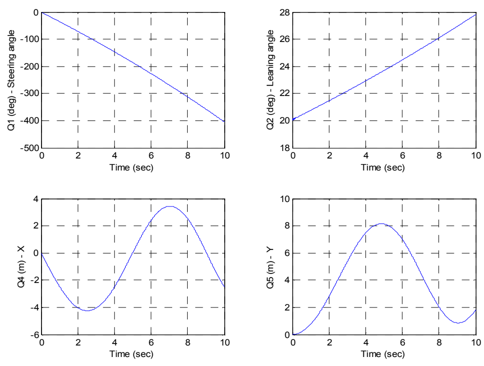

Fig.2 Generalized Coordinates q_i of a One-Wheel Robot

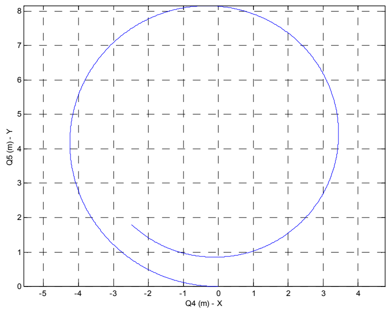

Fig.3 Position of a Contact Point (q_4 , q_5) of a One-Wheel Robot on X-Y Plane

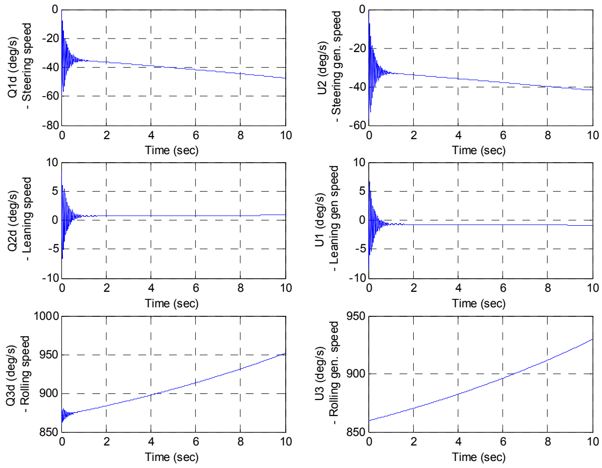

Fig.4 Time Derivative of Generalized Coordinates $\dot{q}_i$ and Generalized Speeds u_i of a One-Wheel Robot

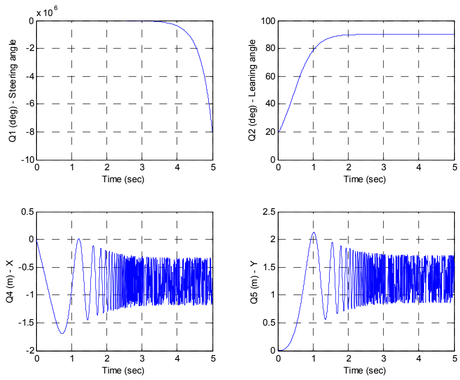

Fig.5 Generalized Coordinates q_i of a Rolling Disk

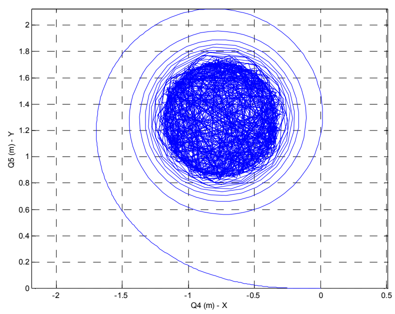

Fig.6 Position of a Contact Point (q_4 , q_5) of a Rolling Disk on X-Y Plane

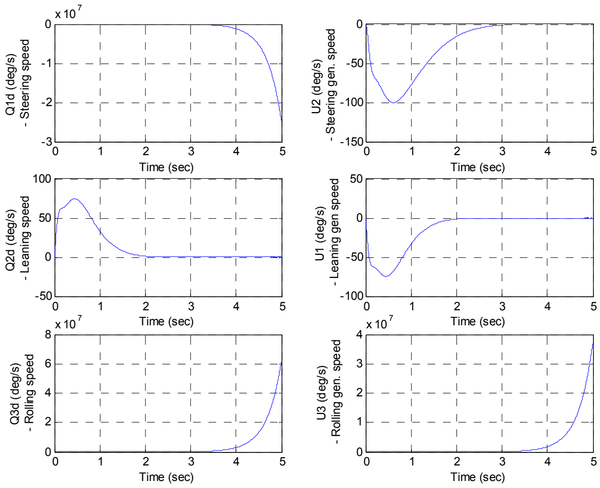

Fig.7 Time Derivative of Generalized Coordinates $\dot{q}_i$ and Generalized Speeds u_i of a Rolling Disk

5. Conclusions and Future Works

The advantages of using Kane's method have showed that it is suitable for dealing with nonholonomic systems. Equations of motion of a one-wheel robot were derived by this method. Numerical simulations are presented to verify validity of the model in agreement with the Lagrange's formulation found in early works by Xu [9]. In future works, the nonlinear controller of the one-wheel robot for following desired paths will be proposed.

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