Extended Kalman Filter with Adaptive Measurement Noise Characteristics for Position Estimation of an Autonomous Vehicle

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Abstract—**This paper proposes the position estimation method of an autonomous vehicle on flat terrain, which based on playback navigation algorithm. The proposed method is sensor fusion using the extended Kalman Filter (EKF) for state estimation from the low-cost Global Positioning System (GPS) receiver and incremental encoder. The Singular Value Decomposition (SVD) is applied to evaluate the adaptive measurement noise covariance in the EKF. This improves the accuracy of estimation to correspond to the errors involved along various portions of the trajectory, instead of using fixed values. The result showed that the proposed method can improve an accuracy of position estimation of autonomous vehicle on flat terrain.**

I. INTRODUCTION

There are various autonomous vehicle navigation methods such as vision based navigation [1], line following using opto-sensor and magnet based navigation [2]. A Playback algorithm [3] is another way of the navigation system that the vehicle can learn course data by only manually driven on a path without environment modified. The playback algorithm can be applied for an autonomous vehicle that regularly works on the repeated or specified path.

This report focuses on integrating data from a low-cost GPS receiver and dead reckoning to construct a reference path of the playback navigation scheme. By considering the steering encoder gives the steering angle data and the wheel encoder provides the traveled distance of the vehicle. The GPS receiver provides global position and orientation of vehicle. However, the GPS data can subject to degradation in the presence of signal blockage, interference and multipath [4].

The extended Kalman filter has been the extensively used in the field of sensor fusion and navigation [5], [6], [7]. The filter is based upon the principle of linearizing the state transition matrix and the observation matrix with Taylor series expansions. The divergence problem can occur if the theoretical and actual behavior of a filter does not agree [8]. This paper intends to improve estimation by using the pre-record GPS data from several receivers. Data sets were clustered and their covariance matrix is evaluated by the SVD technique. Then, the extended Kalman filter was updated with variable covariance noise measurement instead of using a constant matrix. The result shown adaptive sensor noise covariance can minimize the divergence problem which the actual error of sensor approaches a larger bound than the predicted one.

II. METHODOLOGY

A. *Hierarchical Data Clustering*

GPS data are clustered into small group along the path using hierarchy clustering (Agglomerative) by considering the similarity between every pair of objects in the data set. The idea of method is firstly consider each data as a distinct (singleton) cluster and successively merges clusters together until a stopping criterion is satisfied.

Mahalanobis distance is used to find the similarity. Where the distance from a group of values with means (μ) $\mu = (\mu_1, \mu_2, \mu_3,\ldots, \mu_p)^T$ and covariance matrix P for a multivariate vector $x = (x_1, x_2, x_3,\ldots, x_p)^T$. It can define as

$$D_M(x) = \sqrt{(x-\mu)^T P^{-1}(x-\mu)} \; . \tag{1}$$

The objected is grouped into a binary, hierarchical cluster tree by linking the pairs of objects that are in close proximity together with determining the Weighted Pair Group Method Using Arithmetic Average (WPGMA). The newly formed clusters were linked into larger clusters until a hierarchical tree is formed. The distance between this closest pair of groups is compared to the threshold value. If the distance between this closest pair is less than the threshold distance, these groups become linked and were merged into a single group. The clustering is continued until the distance between the closest pair is greater than the threshold, then the clustering stops.

B. *Evaluate of covariance matrix from SVD*

The covariance of two random variables is their tendency to vary together that expressed as (2).

$$\operatorname{cov}(X,Y) = \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{N} \tag{2}$$

Where $\bar{x}$ is mean(X) and $\bar{y}$ is mean(Y). The covariance matrix is a matrix of covariance (C) between elements of a vector, with elements $C_{i,j}$ = cov(i,j).

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Considering the column data matrix A as the sample vectors, the covariance matrix C written as (3).

$$C = \frac{1}{N} A A^T \tag{3}$$

In order to evaluate the covariance matrix of the clustering GPS data set by SVD technique, the data is centered by subtracting sample vectors with the mean of data group. Then, sample vectors were written in matrix form (A matrix) and factored into a set of rotation and a scale as following.

$$A = USV^T \tag{4}$$

Where U is the normalized eigenvectors of the matrix AA^T, V is the normalized eigenvectors of the matrix A^TA. The columns of U and V are called the left- and right-singular vectors of A respectively. And S is the diagonal matrix which the square-roots of the eigenvalues of matrix A^TA and AA^T are the singular values along the diagonal of the S matrix [9]. Since U can be calculated as the eigenvectors of AA^T. So, U matrix is eigenvectors of covariance matrix which are the axes of maximum variance. While S matrix represents in scale factor.

C. Extended Kalman Filter Design

1) System State Equation

The vehicle model, simplified to be a tricycle, is based on the geometry of motion of the Ackerman vehicle, see Fig. 1. The state of the vehicle at the k th sampling time is

$$X_k = [x_k, y_k, \theta_k]^T .$$

When x_k, y_k and θ_k represent the vehicle position origin at the center of rear axle and the heading angle of the vehicle in the reference frame (X_{ref}, Y_{ref}), respectively. The state vector X_k can obtain by assuming the sampling period is small enough, as following:

$$X_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + \Delta x_k \\ y_{k-1} + \Delta y_k \\ \theta_{k-1} + \Delta\theta_k \end{bmatrix} + w_k$$

$$= \begin{bmatrix} x_{k-1} + v_k \Delta t_k \cos(\theta_{k-1} + \frac{\Delta\theta_k}{2}) \\ y_{k-1} + v_k \Delta t_k \sin(\theta_{k-1} + \frac{\Delta\theta_k}{2}) \\ \theta_{k-1} + \Delta\theta_k \\ \end{bmatrix} + w_k \tag{5}$$

Where v_k and θ_{k-1} denote the vehicle velocity and the vehicle heading angle in time interval Δt_k (time step k-1 to k) and w_k is the process noise. Two incremental encoders were attached at steering and rear wheel to measure changing in the steered angle (ϕ_k) and traveled distance (d_k), respectively.

The v_k is product of angular velocity (ω_k) and radius (r_{wheel}) of the moving wheel.

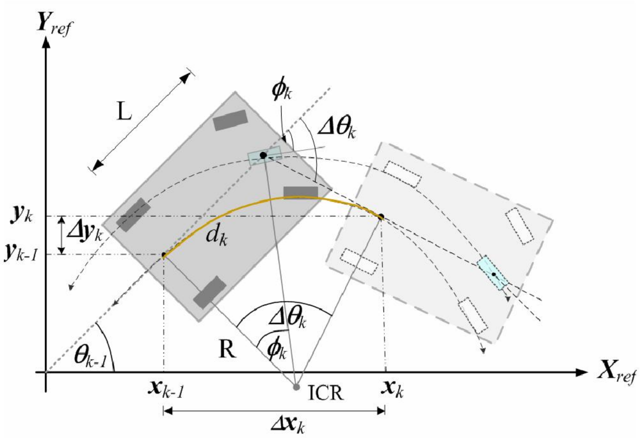


Fig. 1. Variation of the position and orientation after movement with the Ackerman modeling in discrete time.

$$v_k = \omega_k r_{wheel} \tag{6}$$

Since an angular velocity can compute by using pulse from wheel encoder (A_k) that can count in the interval Δt_k, when N is a count number per revolution of encoder.

$$\varpi_k = \frac{2\pi A_k / N}{\Delta t_k} \tag{7}$$

The relation between the steered angle (ϕ_k) and the change in vehicle heading ($\Delta\theta_k$) is shown in (8), which L is the length of the vehicle.

$$\Delta\theta_k = \frac{v_k}{L} \tan(\phi_k) \tag{8}$$

Equation (5) will be used as the system state equation for determine the position and orientation of the vehicle.

2) GPS Measurement Model

Two models of GPS receiver were placed on car. The position measurement (x_{gps}, y_{gps} and θ_{gps}) provided by receiver relates with state vector as below. Where $v_{gps,k}$ is the GPS measurement noise which assumed to be white noise with normal probability distributions. And its covariance obtains from the product of eigenvectors and eigenvalues in SVD step.

$$z_{gps} = \begin{bmatrix} x_{gps,k} \\ y_{gps,k} \\ \theta_{gps,k} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} + v_{gps,k} \tag{9}$$

3) EKF Solution

The state and observer equations must derive to be linear for the extended Kalman filter application as shown in (10).

$$\begin{aligned} X_k &= A(X_{k-1}, u_k) + W w_{k-1} \\ z_k &= H(X_k) + V v_k \end{aligned} \tag{10}$$

Where:

- $X_k \in R^n$ denotes the state vector, z_k is the observer vector.
- $u_k = [v_k, \phi_k]^T$ is the input of the system.

- A and W denote the Jacobian matrix of partial derivatives of the system state with respect to X and w. As following in (11).

$$A = \begin{bmatrix} 1 & 0 & -v_k \Delta t_k \sin(\theta_{k-1} + \frac{\Delta\theta_k}{2}) \\ 0 & 1 & v_k \Delta t_k \cos(\theta_{k-1} + \frac{\Delta\theta_k}{2}) \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- H and V are the Jacobian matrix of partial derivatives of the observer equation (h) with respect to X and v. They are verify

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (12)$$

- The random variables w_k and v_k denote the process and measurement noise with probability distributions as $p(w) \sim N(0,Q)$ and $p(v) \sim N(0,R)$, respectively.

The detailed EKF equations are referenced from [10]. In the predictive phase, the filter uses the odometric data to compute the predictive state every 0.1 seconds. In the update phase, the GPS measurements are used to update the state estimation whenever each new measurement occurs. Since the low-cost GPS which is used in this proposed system is updated at a very low frequency, by assuming that all measurements come in simultaneously could lead to some drawbacks such as low estimate rate and lack of means to deal with unusual noise from an individual measurement [11]. Therefore, by integrating each measurement input immediately as it is obtained, the estimation would be more frequent and accurate. The methodology diagram could be seen in Fig. 2.

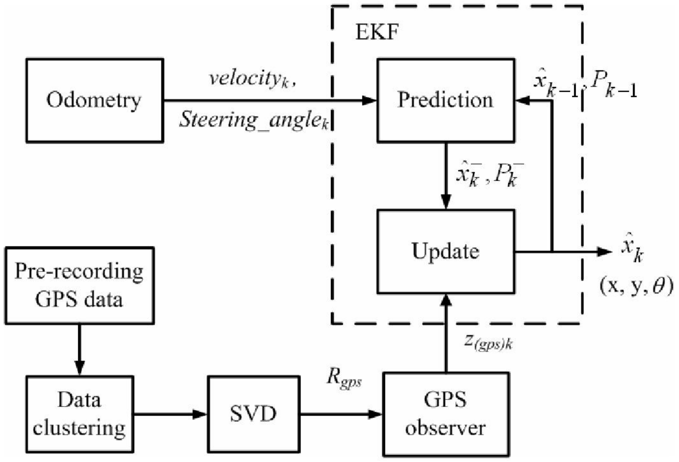


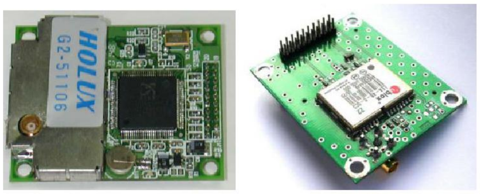
Fig. 2. Architecture of estimate system

III. EXPERIMENT SYSTEM

The experimental vehicle, as seen in Fig. 3, is an internal combustion engine vehicle with rear wheel drive. The vehicle throttle is controlled by a servo motor. The speed of the vehicle is controlled under the assumption that the vehicle velocity and the angular position of a servo motor has a linear relation.

Data from the steering wheel encoder and the rear wheel encoders were sampled at 10 Hz. Two different brands of GPS receiver modules were used in this experiment to compare their different characteristics. As shown in Fig. 4, two HOLUX GM-82 engine boards (4a) and one U-Blox GPS model LEA-4S board (4b) were used. The former device performance specification has position error about 5-25 m CEP and update rate at 1 Hz. The latter device performance specification has position error at 2.5 m CEP and the maximum update rate at 4 Hz. All sensors connect with the main processor (on board computer notebook) via serial and USB. The overall system diagram is shown in Fig.5.

Fig.3. The experimental vehicle with three GPS receivers, the steering encoder and the rear wheel encoder.



(a) (b)

Fig.4. Two models of GPS receiver a) HOLUX GM-82 b) U-Blox LEA-4S

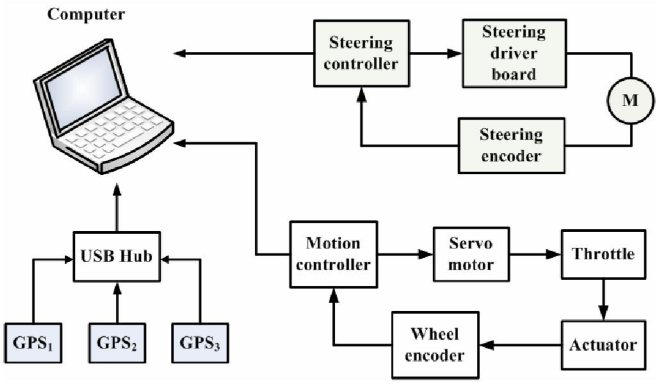


Fig. 5. System diagram

IV. EXPERIMENT RESULTS

The experimental results were obtained by post-processing the actual sensor measurements acquired by driving the experimental vehicle along the oval track around the football field at KMUTT. The sensor data from GPS and encoder were recorded along with the manual control.

Fig.6, illustrates three GPS data sets which were recorded and clustered into 20 groups. Notice that some parts of the record path such as the vertical section are surrounded by high buildings and trees. Thus, the GPS data can be affected by the multipath error and cause the data to be erroneous.

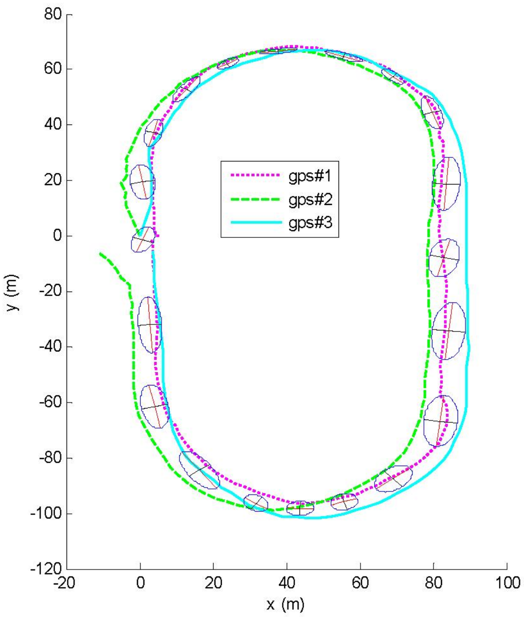


Fig. 6. The elliptic standard deviation of the GPS data after clustered.

The estimate results of three different methods were compared including: the odometry-based (dead reckoning) method, the EKF with predefined bounded errors of GPS data and the proposed method with adaptive noise covariance of GPS data. In Fig. 7, the solid line, star symbol line and dotted line are the GPS position of three GPS receivers. The maximum range between them is about 10 m. The dashed line is the odometry estimation, which is the relative measurement, was greatly affected by the accumulative errors from skidding and slippage of the vehicle.

The bound errors EKF can improve the estimate result by the absolute data from the GPS receiver. However, the noise characteristic of GPS data is different at various locations along the path. By using the maximum bound of error, In Fig. 7, the accuracy of state estimation is reduced as shown in the triangle sign line. The proposed method varies the bound of error in the filter according to different location, thus can increase the estimation accuracy and add robustness to the system in case of noisy measurement as shown in the dash dot line.

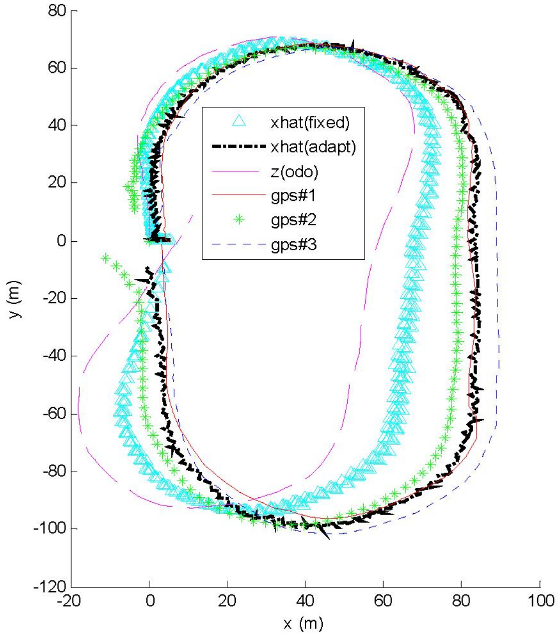


Fig. 7. The experiment results from three methods: odometry, EKF with fixed noise covariance of GPS and EKF with adaptive noise covariance.

Fig. 8, presents the state in x and y direction and orientation of the vehicle with the time step. The proposed estimate results were compared to GPS data and the odometry estimation. In the long term, GPS data provides time-invariant errors that better than the odometry. But in the short term, the odometry measurement provides reliable data more than GPS data. And in the location where GPS variance is large, the estimation will decrease the reliability in GPS data to update state.

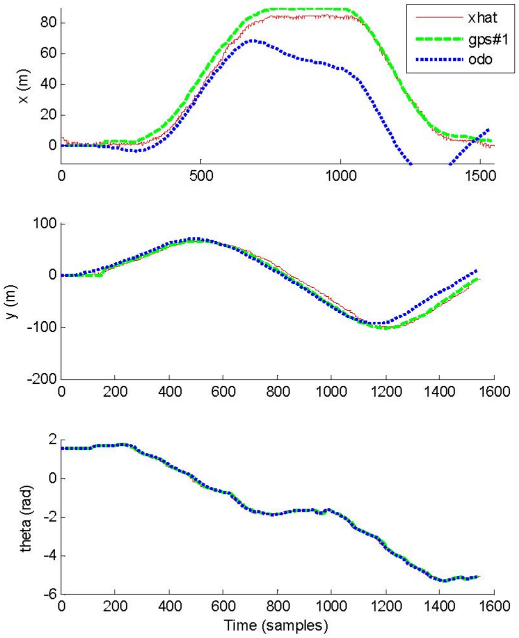


Fig. 8. The experiment results from three methods.

V. CONCLUSION

This paper presented the method that can improve position estimation of an autonomous vehicle. By clustering the pre-recorded GPS data along the path into small groups, the statistical knowledge of the location dependence error of the GPS data can be formed. By finding the covariance matrix using SVD technique, the magnitudes and orientation of errors in each data group can be visualized and then applied to the standard position estimation technique such as EKF.

The pre-processed GPS covariance matrix was used in the measurement update equation of EKF. Therefore, the modified EKF performs state estimation with adaptive sensor noise characteristics. The experiments were performed to compare the performance between three different position estimation methods: the odometry based method, the conventional EKF with bounded error and the proposed method with adaptive GPS covariance.

The experimental results show that the proposed method can handle the GPS location-based error such as multipath. This also improve an accuracy of estimation without use the differential GPS. But the method does not carry out another erroneous type like atmospheric refraction or blockage of satellite signal. In the future, more GPS information (such as number of visible satellites and signal to noise ratio) would be considered to improve adaptive covariance matrix of GPS receivers.

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